ANSWERS

EXERCISE 1.1

1. (i) Neither reflexive nor symmetric nor transitive.

(ii) Neither reflexive nor symmetric but transitive.

(iii) Reflexive and transitive but not symmetric.

(iv) Reflexive, symmetric and transitive.

(v) (a) Reflexive, symmetric and transitive.

(b) Reflexive, symmetric and transitive.

(c) Neither reflexive nor symmetric nor transitive.

(d) Neither reflexive nor symmetric but transitive.

(e) Neither reflexive nor symmetric nor transitive.

3. Neither reflexive nor symmetric nor transitive.

5. Neither reflexive nor symmetric nor transitive.

9. (i) $\{1, 5, 9\}$, (ii) $\{1\}$

12. T_1 is related to T_3.

13. The set of all triangles

14. The set of all lines $y = 2x + c$, $c \in \mathbf{R}$

15. B

16. C

EXERCISE 1.2

1. No

2. (i) Injective but not surjective

(ii) Neither injective nor surjective

(iii) Neither injective nor surjective

(iv) Injective but not surjective

(v) Injective but not surjective

7. (i) One-one and onto

(ii) Neither one-one nor onto.

9. No **10.** Yes **11.** D **12.** A

EXERCISE 1.3

1. $gof = \{(1, 3), (3,1), (4,3)\}$

3. (i) $(gof)(x) = |5|x| - 2|$, $(fog)(x) = |5x - 2|$

(ii) $(g \circ f)(x) = 2x$, $(f \circ g)(x) = 8x$

4. Inverse of f is f itself

5. (i) No, since f is many-one (ii) No, since g is many-one.
(iii) Yes, since h is one-one-onto.

6. f^{-1} is given by $f^{-1}(y) = \frac{2y}{1-y}$, $y \neq 1$ **7.** f^{-1} is given by $f^{-1}(y) = \frac{y-3}{4}$

11. f^{-1} is given by $f^{-1}(a) = 1$, $f^{-1}(b) = 2$ and $f^{-1}(c) = 3$.

13. (C) **14.** (B)

EXERCISE 1.4

1. (i) No (ii) Yes (iii) Yes (iv) Yes (v) Yes

2. (i) $*$ is binary but neither commutative nor associative
(ii) $*$ is binary, commutative but not associative
(iii) $*$ is binary, both commutative and associative
(iv) $*$ is binary, commutative but not associative
(v) $*$ is binary but neither commutative nor associative
(vi) $*$ not binary

3.

$\wedge$	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

4. (i) $(2 * 3) * 4 = 1$ and $2 * (3 * 4) = 1$ (ii) Yes (iii) 1

5. Yes

6. (i) $5 * 7 = 35$, $20 * 16 = 80$ (ii) Yes (iii) Yes (iv) 1 (v) 1

7. No **8.** $*$ is both commutative and associative; $*$ does not have any identity in **N**

9. (ii) , (iv), (v) are commutative; (v) is associative. **10.** (v)

11. Identity element does not exist.

12. (ii) False (ii) True **13.** B

Miscellaneous Exercise on Chapter 1

1. $g(y) = \frac{y-7}{10}$ **2.** The inverse of f is f itself

3. $x^4 - 6x^3 + 10x^2 - 3x$ **8.** No **10.** $n!$

11. (i) $F^{-1} = \{(3, a), (2, b), (1, c)\}$, (ii) F^{-1} does not exist **12.** No

15. Yes **16.** A **17.** B **18.** No

19. B

EXERCISE 2.1

1. $\frac{-\pi}{6}$ **2.** $\frac{\pi}{6}$ **3.** $\frac{\pi}{6}$ **4.** $\frac{-\pi}{3}$

5. $\frac{2\pi}{3}$ **6.** $-\frac{\pi}{4}$ **7.** $\frac{\pi}{6}$ **8.** $\frac{\pi}{6}$

9. $\frac{3\pi}{4}$ **10.** $\frac{-\pi}{4}$ **11.** $\frac{3\pi}{4}$ **12.** $\frac{2\pi}{3}$

13. B **14.** B

EXERCISE 2.2

5. $\frac{1}{2}\tan^{-1} x$ **6.** $\frac{\pi}{2} - \sec^{-1} x$ **7.** $\frac{x}{2}$ **8.** $\frac{\pi}{4} - x$

9. $\sin^{-1}\frac{x}{a}$ **10.** $3\tan^{-1}\frac{x}{a}$ **11.** $\frac{\pi}{4}$ **12.** 0

13. $\frac{x+y}{1-xy}$ **14.** $\frac{1}{5}$ **15.** $\pm\frac{1}{\sqrt{2}}$ **16.** $\frac{\pi}{3}$

17. $\frac{-\pi}{4}$ **18.** $\frac{17}{6}$ **19.** B **20.** D

21. B

Miscellaneous Exercise on Chapter 2

1. $\frac{\pi}{6}$ **2.** $\frac{\pi}{6}$ **13.** $x = n\pi + \frac{\pi}{4}, n \in \not{Z}$ **14.** $x = \frac{1}{\sqrt{3}}$

15. D **16.** C **17.** C

EXERCISE 3.1

1. (i) 3×4 (ii) 12 (iii) $19, 35, -5, 12, \frac{5}{2}$

2. $1 \times 24, 2 \times 12, 3 \times 8, 4 \times 6, 6 \times 4, 8 \times 3, 12 \times 2, 24 \times 1$; $1 \times 13, 13 \times 1$

3. $1 \times 18, 2 \times 9, 3 \times 6, 6 \times 3, 9 \times 2, 18 \times 1$; $1 \times 5, 5 \times 1$

4. (i) $\begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}$

5. (i) $\begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$

6. (i) $x = 1, \quad y = 4, \quad z = 3$

(ii) $x = 4, \quad y = 2, \quad z = 0$ or $x = 2, \quad y = 4, \quad z = 0$

(iii) $x = 2, \quad y = 4, \quad z = 3$

7. $a = 1, b = 2, c = 3, d = 4$

8. C **9.** B **10.** D

EXERCISE 3.2

1. (i) $A + B = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$ (ii) $A - B = \begin{bmatrix} 1 & 1 \\ 5 & -3 \end{bmatrix}$

(iii) $3A - C = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$ (iv) $AB = \begin{bmatrix} -6 & 26 \\ -1 & 19 \end{bmatrix}$ (v) $BA = \begin{bmatrix} 11 & 10 \\ 11 & 2 \end{bmatrix}$

2. (i) $\begin{bmatrix} 2a & 2b \\ 0 & 2a \end{bmatrix}$ (ii) $\begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$

(iii) $\begin{bmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}$ (iv) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

3. (i) $\begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix}$ (iii) $\begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$

(iv) $\begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$ (v) $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$ (vi) $\begin{bmatrix} 14 & -6 \\ 4 & 5 \end{bmatrix}$

4. $A+B=\begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix}$, $B-C=\begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$

5. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ **6.** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

7. (i) $X=\begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$, $Y=\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$ (ii) $X=\begin{bmatrix} \frac{2}{5} & \frac{-12}{5} \\ \frac{-11}{5} & 3 \end{bmatrix}$, $Y=\begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$

8. $X=\begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$ **9.** $x=3, y=3$ **10.** $x=3, y=6, z=9, t=6$

11. $x=3, y=-4$ **12.** $x=2, y=4, w=3, z=1$

15. $\begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$ **17.** $k=1$

19. (a) ₹15000, ₹15000 (b) ₹5000, ₹25000

20. ₹20160 **21.** A **22.** B

EXERCISE 3.3

1. (i) $\begin{bmatrix} 5 & \frac{1}{2} & -1 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ (iii) $\begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}$

4. $\begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$ **9.** $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

10. (i) $A = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(iii) $A = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ \frac{-5}{2} & 0 & 3 \\ \frac{-3}{2} & -3 & 0 \end{bmatrix}$ (iv) $A = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$

11. A **12.** B

EXERCISE 3.4

1. $\begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{-2}{5} & \frac{1}{5} \end{bmatrix}$ **2.** $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ **3.** $\begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$

4. $\begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$ **5.** $\begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$ **6.** $\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

7. $\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$ **8.** $\begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$ **9.** $\begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$

10. $\begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix}$ **11.** $\begin{bmatrix} -1 & 3 \\ \frac{-1}{2} & 1 \end{bmatrix}$ **12.** Inverse does not exist.

13. $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ **14.** Inverse does not exist.

15. $\begin{bmatrix} \frac{-2}{5} & 0 & \frac{3}{5} \\ \frac{-1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{bmatrix}$ **16.** $\begin{bmatrix} 1 & \frac{-2}{5} & \frac{-3}{5} \\ \frac{-2}{5} & \frac{4}{25} & \frac{11}{25} \\ \frac{-3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$ **17.** $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

18. D

Miscellaneous Exercise on Chapter 3

6. $x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{6}}, z = \pm\frac{1}{\sqrt{3}}$

7. $x = -1$ **9.** $x = \pm 4\sqrt{3}$

10. (a) Total revenue in the market - I = ₹46000

Total revenue in the market - II = ₹53000

(b) ₹15000, ₹17000

11. $X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$ **13.** C **14.** B **15.** C

EXERCISE 4.1

1. (i) 18 **2.** (i) 1, (ii) $x^3 - x^2 + 2$

5. (i) -12, (ii) 46, (iii) 0, (iv) 5 **6.** 0

7. (i) $x = \pm\sqrt{3}$, (ii) $x = 2$ **8.** (B)

EXERCISE 4.2

15. C **16.** C

EXERCISE 4.3

1. (i) $\frac{15}{2}$, (ii) $\frac{47}{2}$, (iii) 15

3. (i) 0, 8, (ii) 0, 8 **4.** (i) $y = 2x$, (ii) $x - 3y = 0$ **5.** (D)

EXERCISE 4.4

1. (i) $M_{11} = 3, M_{12} = 0, M_{21} = -4, M_{22} = 2, A_{11} = 3, A_{12} = 0, A_{21} = 4, A_{22} = 2$

(ii) $M_{11} = d, M_{12} = b, M_{21} = c, M_{22} = a$

$A_{11} = d, A_{12} = -b, A_{21} = -c, A_{22} = a$

2. (i) $M_{11} = 1, M_{12} = 0, M_{13} = 0, M_{21} = 0, M_{22} = 1, M_{23} = 0, M_{31} = 0, M_{32} = 0, M_{33} = 1,$

$A_{11} = 1, A_{12} = 0, A_{13} = 0, A_{21} = 0, A_{22} = 1, A_{23} = 0, A_{31} = 0, A_{32} = 0, A_{33} = 1$

(ii) $M_{11} = 11, M_{12} = 6, M_{13} = 3, M_{21} = -4, M_{22} = 2, M_{23} = 1, M_{31} = -20, M_{32} = -13, M_{33} = 5$

$A_{11} = 11, A_{12} = -6, A_{13} = 3, A_{21} = 4, A_{22} = 2, A_{23} = -1, A_{31} = -20, A_{32} = 13, A_{33} = 5$

3. 7 **4.** $(x - y)(y - z)(z - x)$ **5.** (D)

EXERCISE 4.5

1. $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$ **2.** $\begin{bmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{bmatrix}$ **5.** $\frac{1}{14}\begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$

6. $\frac{1}{13}\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$ **7.** $\frac{1}{10}\begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$ **8.** $\frac{-1}{3}\begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$

9. $\frac{-1}{3}\begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$ **10.** $\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ **11.** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

13. $\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ **14.** $a = -4, b = 1$ **15.** $A^{-1} = \frac{1}{11}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$

16. $\frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$ **17.** B **18.** B

EXERCISE 4.6

1. Consistent **2.** Consistent **3.** Inconsistent

4. Consistent **5.** Inconsistent **6.** Consistent

7. $x = 2, y = -3$ **8.** $x = \frac{-5}{11}, y = \frac{12}{11}$ **9.** $x = \frac{-6}{11}, y = \frac{-19}{11}$

10. $x = -1, y = 4$ **11.** $x = 1, y = \frac{1}{2}, z = \frac{-3}{2}$

12. $x = 2, y = -1, z = 1$ **13.** $x = 1, y = 2, z = -1$

14. $x = 2, y = 1, z = 3$

15. $\begin{matrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{matrix}$, $x = 1, y = 2, z = 3$

16. cost of onions per kg = ₹ 5
cost of wheat per kg = ₹ 8
cost of rice per kg = ₹ 8

Miscellaneous Exercise on Chapter 4

3. 1 **5.** $x = \frac{-a}{3}$ **7.** $\begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

9. $-2(x^3 + y^3)$ **10.** xy **16.** $x = 2, y = 3, z = 5$

17. A **18.** A **19.** D

EXERCISE 5.1

2. f is continuous at $x = 3$

3. (a), (b), (c) and (d) are all continuous functions

5. f is continuous at $x = 0$ and $x = 2$; Not continuous at $x = 1$

6. Discontinuous at $x = 2$ **7.** Discontinuous at $x = 3$

8. Discontinuous at $x = 0$ **9.** No point of discontinuity

10. No point of discontinuity **11.** No point of discontinuity

12. f is discontinuous at $x = 1$ **13.** f is not continuous at $x = 1$

14. f is not continuous at $x = 1$ and $x = 3$

15. $x = 1$ is the only point of discontinuity

16. Continuous **17.** $a = b + \frac{2}{3}$

18. For no value of λ, f is continuous at $x = 0$ but f is continuous at $x = 1$ for any value of λ.

20. f is continuous at $x = \pi$ **21.** (a), (b) and (c) are all continuous

22. Cosine function is continuous for all $x \in \mathbf{R}$; cosecant is continuous except for $x = n\pi$, $n \in \mathbf{Z}$; secant is continuous except for $x = (2n+1)\frac{\pi}{2}$, $n \in \mathbf{Z}$ and cotangent function is continuous except for $x = n\pi$, $n \in \mathbf{Z}$

23. There is no point of discontinuity.

24. Yes, f is continuous for all $x \in \mathbf{R}$ **25.** f is continuous for all $x \in \mathbf{R}$

26. $k = 6$ **27.** $k = \frac{3}{4}$ **28.** $k = \frac{-2}{\pi}$

29. $k = \frac{9}{5}$ **30.** $a = 2, b = 1$

34. There is no point of discontinuity.

EXERCISE 5.2

1. $2x \cos(x^2 + 5)$ **2.** $-\cos x \sin(\sin x)$ **3.** $a \cos(ax + b)$

4. $\frac{\sec(\tan\sqrt{x}).\tan(\tan\sqrt{x}).\sec^2\sqrt{x}}{2\sqrt{x}}$

5. $a \cos(ax + b) \sec(cx + d) + c \sin(ax + b) \tan(cx + d) \sec(cx + d)$

6. $10x^4 \sin x^5 \cos x^5 \cos x^3 - 3x^2 \sin x^3 \sin^2 x^5$

7. $\dfrac{-2\sqrt{2}\,x}{\sin x^2\sqrt{\sin 2x^2}}$ **8.** $-\dfrac{\sin\sqrt{x}}{2\sqrt{x}}$

EXERCISE 5.3

1. $\dfrac{\cos x-2}{3}$ **2.** $\dfrac{2}{\cos y-3}$ **3.** $-\dfrac{a}{2by+\sin y}$

4. $\dfrac{\sec^2 x-y}{x+2y-1}$ **5.** $-\dfrac{(2x+y)}{(x+2y)}$ **6.** $-\dfrac{(3x^2+2xy+y^2)}{(x^2+2xy+3y^2)}$

7. $\dfrac{y\sin xy}{\sin 2y-x\sin xy}$ **8.** $\dfrac{\sin 2x}{\sin 2y}$ **9.** $\dfrac{2}{1+x^2}$ **10.** $\dfrac{3}{1+x^2}$

11. $\dfrac{2}{1+x^2}$ **12.** $\dfrac{-2}{1+x^2}$ **13.** $\dfrac{-2}{1+x^2}$ **14.** $\dfrac{2}{\sqrt{1-x^2}}$

15. $-\dfrac{2}{\sqrt{1-x^2}}$

EXERCISE 5.4

1. $\dfrac{e^x(\sin x-\cos x)}{\sin^2 x}$, $x\neq n\pi$, $n\in\mathbf{Z}$ **2.** $\dfrac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}$, $x\in(-1,1)$

3. $3x^2e^{x^3}$ **4.** $-\dfrac{e^{-x}\cos(\tan^{-1}e^{-x})}{1+e^{-2x}}$

5. $-e^x\tan e^x$, $e^x\neq(2n+1)\dfrac{\pi}{2}$, $n\in\mathbf{N}$ **6.** $e^x+2x^{e^{x^2}}+3x^2e^{x^3}+4x^3e^{x^4}+5x^4e^{x^5}$

7. $\dfrac{e^{\sqrt{x}}}{4\sqrt{xe^{\sqrt{x}}}}$, $x>0$ **8.** $\dfrac{1}{x\log x}$, $x>1$

9. $-\dfrac{(x\sin x\cdot\log x+\cos x)}{x(\log x)^2}$, $x>0$ **10.** $-\dfrac{1}{x}+e^x\ \sin(\log x+e^x)$, $x>0$

EXERCISE 5.5

1. $-\cos x \cos 2x \cos 3x\,[\tan x + 2\tan 2x + 3\tan 3x]$

2. $\frac{1}{2}\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}\left[\frac{1}{x-1}+\frac{1}{x-2}-\frac{1}{x-3}-\frac{1}{x-4}-\frac{1}{x-5}\right]$

3. $(\log x)^{\cos x}\left[\frac{\cos x}{x\log x}-\sin x\log(\log x)\right]$

4. $x^x(1+\log x)-2^{\sin x}\cos x\log 2$

5. $(x+3)(x+4)^2(x+5)^3(9x^2+70x+133)$

6. $\left(x+\frac{1}{x}\right)^x\left[\frac{x^2-1}{x^2+1}+\log\left(x+\frac{1}{x}\right)\right]+x^{1+\frac{1}{x}}\left(\frac{x+1-\log x}{x^2}\right)$

7. $(\log x)^{x-1}[1+\log x\,.\,\log(\log x)]+2x^{\log x-1}\,.\,\log x$

8. $(\sin x)^x(x\cot x+\log\sin x)+\frac{1}{2}\frac{1}{\sqrt{x-x^2}}$

9. $x^{\sin x}\left[\frac{\sin x}{x}+\cos x\log x\right]+(\sin x)^{\cos x}[\cos x\cot x-\sin x\log\sin x]$

10. $x^{x\cos x}[\cos x\,.\,(1+\log x)-x\sin x\log x]-\frac{4x}{(x^2-1)^2}$

11. $(x\cos x)^x[1-x\tan x+\log(x\cos x)]+(x\sin x)^{\frac{1}{x}}\left[\frac{x\cot x+1-\log(x\sin x)}{x^2}\right]$

12. $-\frac{yx^{y-1}+y^x\log y}{x^y\log x+xy^{x-1}}$

13. $\frac{y}{x}\left(\frac{y-x\log y}{x-y\log x}\right)$

14. $\frac{y\tan x+\log\cos y}{x\tan y+\log\cos x}$

15. $\frac{y(x-1)}{x(y+1)}$

16. $(1+x)(1+x^2)(1+x^4)(1+x^8)\left[\frac{1}{1+x}+\frac{2x}{1+x^2}+\frac{4x^3}{1+x^4}+\frac{8x^7}{1+x^8}\right]$; $f'(1)=120$

17. $5x^4-20x^3+45x^2-52x+11$

EXERCISE 5.6

1. t^2 2. $\frac{b}{a}$ 3. $-4\sin t$ 4. $-\frac{1}{t^2}$

5. $\frac{\cos\theta - 2\cos 2\theta}{2\sin 2\theta - \sin\theta}$ **6.** $-\cot\frac{\theta}{2}$ **7.** $-\cot 3t$ **8.** $\tan t$

9. $\frac{b}{a}\operatorname{cosec}\theta$ **10.** $\tan\theta$

EXERCISE 5.7

1. 2 **2.** $380\, x^{18}$ **3.** $-x\cos x - 2\sin x$

4. $-\frac{1}{x^2}$ **5.** $x(5 + 6\log x)$ **6.** $2e^x(5\cos 5x - 12\sin 5x)$

7. $9\, e^{6x}(3\cos 3x - 4\sin 3x)$ **8.** $-\frac{2x}{(1+x^2)^2}$

9. $-\frac{(1+\log x)}{(x\log x)^2}$ **10.** $-\frac{\sin(\log x) + \cos(\log x)}{x^2}$

12. $-\cot y \operatorname{cosec}^2 y$

Miscellaneous Exercise on Chapter 5

1. $27(3x^2 - 9x + 5)^8(2x - 3)$ **2.** $3\sin x\cos x(\sin x - 2\cos^4 x)$

3. $(5x)^{3\cos 2x}\left[\frac{3\cos 2x}{x} - 6\sin 2x\log 5x\right]$

4. $\frac{3}{2}\sqrt{\frac{x}{1-x^3}}$ **5.** $-\left[\frac{1}{\sqrt{4-x^2}\sqrt{2x+7}} + \frac{\cos^{-1}\frac{x}{2}}{(2x+7)^{\frac{3}{2}}}\right]$

6. $\frac{1}{2}$ **7.** $(\log x)^{\log x}\left[\frac{1}{x} + \frac{\log(\log x)}{x}\right],\ x > 1$

8. $(a\sin x - b\cos x)\sin(a\cos x + b\sin x)$

9. $(\sin x - \cos x)^{\sin x - \cos x}(\cos x + \sin x)(1 + \log(\sin x - \cos x)),\ \sin x > \cos x$

10. $x^x(1 + \log x) + ax^{a-1} + a^x\log a$

11. $x^{x^2-3}\left[\frac{x^2-3}{x} + 2x\log x\right] + (x-3)^{x^2}\left[\frac{x^2}{x-3} + 2x\log(x-3)\right]$

12. $\frac{6}{5}\cot\frac{t}{2}$ **13.** 0 **17.** $\frac{\sec^3 t}{at}, 0 < t < \frac{\pi}{2}$

EXERCISE 6.1

1. (a) 6π cm^2/cm (b) 8π cm^2/cm

2. $\frac{8}{3}$ cm^2/s **3.** 60π cm^2/s **4.** 900 cm^3/s

5. 80π cm^2/s **6.** 1.4π cm/s

7. (a) –2 cm/min (b) 2 cm^2/min

8. $\frac{1}{\pi}$ cm/s **9.** 400π cm^3/cm **10.** $\frac{8}{3}$ cm/s

11. (4, 11) and $\left(-4, \frac{-31}{3}\right)$ **12.** 2π cm^3/s

13. $\frac{27}{8}\pi(2x+1)^2$ **14.** $\frac{1}{48\pi}$ cm/s **15.** ₹ 20.967

16. ₹ 208 **17.** B **18.** D

EXERCISE 6.2

4. (a) $\left(\frac{3}{4}, \infty\right)$ (b) $\left(-\infty, \frac{3}{4}\right)$

5. (a) $(-\infty, -2)$ and $(3, \infty)$ (b) $(-2, 3)$

6. (a) decreasing for $x < -1$ and increasing for $x > -1$

(b) decreasing for $x > -\frac{3}{2}$ and increasing for $x < -\frac{3}{2}$

(c) increasing for $-2 < x < -1$ and decreasing for $x < -2$ and $x > -1$

(d) increasing for $x < -\frac{9}{2}$ and decreasing for $x > -\frac{9}{2}$

(e) increasing in (1, 3) and (3, ∞), decreasing in (– ∞, –1) and (– 1, 1).

8. $0 < x < 1$ and $x > 2$ **12.** A, B

13. D **14.** $a > -2$ **19.** D

EXERCISE 6.3

1. 764 **2.** $\frac{-1}{64}$ **3.** 11 **4.** 24

5. 1 **6.** $\frac{-a}{2b}$ **7.** (3, – 20) and (–1, 12)

8. (3, 1) **9.** (2, – 9)

10. (i) $y + x + 1 = 0$ and $y + x - 3 = 0$

11. No tangent to the curve which has slope 2.

12. $y = \frac{1}{2}$ **13.** (i) $(0, \pm 4)$ (ii) $(\pm 3, 0)$

14. (i) Tangent: $10x + y = 5$; Normal: $x - 10y + 50 = 0$

(ii) Tangent: $y = 2x + 1$; Normal: $x + 2y - 7 = 0$

(iii) Tangent: $y = 3x - 2$; Normal: $x + 3y - 4 = 0$

(iv) Tangent: $y = 0$; Normal: $x = 0$

(v) Tangent: $x + y - \sqrt{2} = 0$; Normal $x = y$

15. (a) $y - 2x - 3 = 0$ (b) $36y + 12x - 227 = 0$

17. (0, 0), (3, 27) **18.** (0, 0), (1, 2), (–1, –2)

19. $(1, \pm 2)$ **20.** $2x + 3my - am^2(2 + 3m^2) = 0$

21. $x + 14y - 254 = 0$, $x + 14y + 86 = 0$

22. $ty = x + at^2$, $y = -tx + 2at + at^3$

24. $\frac{x x_0}{a^2} - \frac{y y_0}{b^2} = 1$, $\frac{y - y_0}{a^2 y_0} + \frac{x - x_0}{b^2 x_0} = 0$

25. $48x - 24y = 23$ **26.** D **27.** A

EXERCISE 6.4

1. (i) 5.03 (ii) 7.035 (iii) 0.775

(iv) 0.208 (v) 0.999 (vi) 1.968

(vii) 2.962 (viii) 3.996 (ix) 3.009

(x) 20.025 (xi) 0.060 (xii) 2.948

(xiii) 3.004 (xiv) 7.904 (xv) 2.001

2. 28.21 **3.** -34.995 **4.** $0.03\ x^3\ m^3$

5. $-0.12\ x^2\ m^2$ **6.** $3.92\ \pi\ m^3$ **7.** $2.16\ \pi\ m^3$

8. D **9.** C

EXERCISE 6.5

1. (i) Minimum Value = 3 (ii) Minimum Value = – 2

(iii) Maximum Value = 10 (iv) Neither minimum nor maximum value

2. (i) Minimum Value = – 1; No maximum value

(ii) Maximum Value = 3; No minimum value

(iii) Minimum Value = 4; Maximum Value = 6

(iv) Minimum Value = 2; Maximum Value = 4

(v) Neither minimum nor Maximum Value

3. (i) local minimum at $x = 0$, local minimum value = 0

(ii) local minimum at $x = 1$, local minimum value = – 2

local maximum at $x = -1$, local maximum value = 2

(iii) local maximum at $x = \frac{\pi}{4}$, local maximum value = $\sqrt{2}$

(iv) local maximum at $x = \frac{3\pi}{4}$, local maximum value = $\sqrt{2}$

local minimum at $x = \frac{7\pi}{4}$, local minimum value = $-\sqrt{2}$

(v) local maximum at $x = 1$, local maximum value = 19

local minimum at $x = 3$, local minimum value = 15

(vi) local minimum at $x = 2$, local minimum value = 2

(vii) local maximum at $x = 0$, local maximum value $= \frac{1}{2}$

(viii) local maximum at $x = \frac{2}{3}$, local maximum value $= \frac{2\sqrt{3}}{9}$

5. (i) Absolute minimum value $= -8$, absolute maximum value $= 8$

(ii) Absolute minimum value $= -1$, absolute maximum value $= \sqrt{2}$

(iii) Absolute minimum value $= -10$, absolute maximum value $= 8$

(iv) Absolute minimum value $= 19$, absolute maximum value $= 3$

6. Maximum profit = 113 unit.

7. Minima at $x = 2$, minimum value $= -39$, Maxima at $x = 0$, maximum value $= 25$.

8. At $x = \frac{\pi}{4}$ and $\frac{5\pi}{4}$ **9.** Maximum value $= \sqrt{2}$

10. Maximum at $x = 3$, maximum value 89; maximum at $x = -2$, maximum value = 139

11. $a = 120$

12. Maximum at $x = 2\pi$, maximum value $= 2\pi$; Minimum at $x = 0$, minimum value $= 0$

13. 12, 12 **14.** 45, 15 **15.** 25, 10 **16.** 8, 8

17. 3 cm **18.** $x = 5$ cm

21. radius $= \left(\frac{50}{\pi}\right)^{\frac{1}{3}}$ cm and height $= 2\left(\frac{50}{\pi}\right)^{\frac{1}{3}}$ cm

22. $\frac{112}{\pi+4}$ cm, $\frac{28\pi}{\pi+4}$ cm **27.** A **28.** D **29.** C

Miscellaneous Exercise on Chapter 6

1. (a) 0.677 (b) 0.497

3. $b\sqrt{3}$ cm²/s **4.** $x + y - 3 = 0$

6. (i) $0 \le x \le \frac{\pi}{2}$ and $\frac{3\pi}{2} < x < 2\pi$ (ii) $\frac{\pi}{2} < x < \frac{3\pi}{2}$

7. (i) $x < -1$ and $x > 1$ (ii) $-1 < x < 1$

8. $\frac{3\sqrt{3}}{4}ab$ **9.** Rs 1000

11. length $= \frac{20}{\pi+4}$ m, breadth $= \frac{10}{\pi+4}$ m

13. (i) local maxima at $x = \frac{2}{7}$ (ii) local minima at $x = 2$

(iii) point of inflection at $x = -1$

14. Absolute maximum $= \frac{5}{4}$, Absolute minimum $= 1$

17. $\frac{4\pi R^3}{3\sqrt{3}}$ **19.** A **20.** B **21.** A

22. B **23.** A **24.** A

SUPPLEMENTARY MATERIAL

CHAPTER 5

Theorem 5 (To be on page 173 under the heading Theorem 5)

(i) Derivative of Exponential Function $f(x) = e^x$.

If $f(x) = e^x$, then

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{e^{x + \Delta x} - e^x}{\Delta x}$$

$$= e^x \cdot \lim_{\Delta x \to 0} \frac{e^{\Delta x} - 1}{\Delta x}$$

$$= e^x \cdot 1 \quad \left[\text{since } \lim_{h \to 0} \frac{e^h - 1}{h} = 1\right]$$

Thus, $\frac{d}{dx}(e^x) = e^x$.

(ii) Derivative of logarithmic function $f(x) = \log_e x$.

If $f(x) = \log_e x$, then

$$f'(x) = \lim_{\Delta x \to 0} \frac{\log_e (x + \Delta x) - \log_e x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\log_e \left(1 + \frac{\Delta x}{x}\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{1}{x} \frac{\log_e \left(1 + \frac{\Delta x}{x}\right)}{\frac{\Delta x}{x}}$$

$$= \frac{1}{x} \quad \left[\text{since } \lim_{h \to 0} \frac{\log_e (1 + h)}{h} = 1\right]$$

Thus, $\frac{d}{dx} \log_e x = \frac{1}{x}$.